Math 53: Multivariable Calculus **Sections 102, 103**

Worksheet for 2020-04-27

Conceptual Review

Question 1. When computing the flux of a vector field through a surface $\mathbf{r}(u, v)$, how might you decide whether to use $\mathbf{r}_u \times \mathbf{r}_v$ or $\mathbf{r}_v \times \mathbf{r}_u$? (How are these vectors related?)

Problems

Problem 1. Let S be the surface $z = x^2 + 4y^2 - 4$, $z \leq 0$ oriented downwards (i.e. negatively). Compute $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$ where

$$\mathbf{F} = (y \log_2(x^2 + 4y^2 + z^2) + 3x^2y^2\cos(x^3))\mathbf{i} + (-3x + 2y\sin(x^3))\mathbf{j} + (e^{yz}\arctan(y^{x^2+1}))\mathbf{k}.$$

Problem 2. We will do this problem on Wednesday. Throughout this problem, let H denote the plane $z = 2x + 4$.

(a) Let $\mathbf{F} = \langle 3yz, xz, xy - yz \rangle$. Show that if C is any oriented simple closed curve contained in the plane H, then $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$, regardless of C.

(b) Let $\mathbf{G} = \langle x^2y - y, 0, y^3/6 \rangle$. If we let D to be any simple closed curve contained in the plane H which is oriented *counterclockwise* when viewed from above, find the maximum possible value of the integral $\int_D \mathbf{G} \cdot d\mathbf{r}$.

Problem 3. Use the divergence theorem to compute the volume enclosed by the surface obtained by rotating the curve $\langle \cos t, 0, \sin(2t) \rangle$ $(-\pi/2 \leq t \leq \pi/2)$ around the z-axis.

Problem 4. We will do this problem on Wednesday. On the previous worksheet, we did the following problems:

(a) Compute the flux of the vector field $\mathbf{F} = \langle x, y, z \rangle$ through the sphere $x^2 + y^2 + z^2 = 9$ oriented outwards.

(b) Compute the flux of the vector field $\langle 0, 0, 4 - z^2 \rangle$ outwards through the closed cylinder with lateral side $x^2 + y^2 = 10$ and lids $z = 0$ and $z = 2$.

Now do these problems again using the divergence theorem, and check that you get the same answers!